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ACOUSTIC PHASED ARRAYS FOR THE DETECTION OF NUCLEAR BURSTS IN THE ATMOSPHERE

T. F. Burke

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T. F. Burke

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PREFACE

This Memorandum was prepared for the Vela Analysis project under the Advanced Research Projects Agency's contract with RAND. The project is a broad study of systems to detect above-ground nuclear bursts. This Memorandum discusses the possible use of phased arrays in detecting the acoustic signal from atmospheric nuclear bursts and as such should contribute to ARPA's R & D program in this field.

SUMMARY

An acoustic phased array serves to improve both the signal-to-turbulent-noise ratio and the signal-to-acoustic-noise ratio. The improvement in acoustic signal-to-noise ratio is, for isotropic noise, the array gain--which is calculable and is discussed herein. The improvement in the turbulent signal-to-noise ratio cannot be calculated, in view of the unknown correlation of noise in single sensors. However, it is thought likely that this improvement will be at least as much as the acoustic improvement--especially at the higher frequencies of greatest interest for small yields.

This Memorandum considers, for simple unshaded square plane arrays, the dependence of array gain upon frequency, azimuth, size of array, number of sensors in the array, and uncertainty in the local velocity of sound. It is shown that an array of 16 sensors at the lattice points of a 1/2 x 1/2 mile square offers a frequency-dependent gain which rises to nearly 10 db at 0.5 cps for any azimuth and for any sound speed in the range of 1000 to 1200 ft/sec. Such an array appears to be suitable for the detection of acoustic signals from atmospheric nuclear blasts.

CONTENTS

LIST OF FIGURES

I. INTRODUCTION

The principal noise which interferes with the reception of very low frequency (below 1 cps) acoustic signals from nuclear bursts in the atmosphere is that which is caused by local atmospheric turbulence passing over the microphone. Such turbulence, which gives rise to a fluctuating pressure, leads to a wide-band noise output which cannot be distinguished, in a single microphone, from noise which might have arrived acoustically. Even at selected low-wind sites this noise from turbulence is many db above true acoustic noise, and is many score db above theoretical thermal noise.

However, because this turbulent noise does not propagate acoustically, the turbulent noises observed in two microphones which are separated by what would be only a fraction of an acoustic wavelength are usually little correlated. Indeed the correlation is that associated with the scale of the atmospheric turbulence, and much of the scale is less than 1000 ft, whereas the acoustic wavelengths of interest are longer than 1000 ft. Consequently, the VLF signal-to-turbulent-noise ratio can be improved by spatial integration (i.e., by summing signals from numerous receivers).

The summing can be done in passive acoustic networks so as to avoid the cost of installing a large number of microphones and their associated electronics. In practice a pipe array is used to sum about 100 signals introduced through small holes distributed along a line about 1000 ft long. If the turbulence were wholly uncorrelated among the 100 sample points, the signal-to-turbulent-noise ratio would be improved 20 db; most of this 20 db is actually obtained over most of the frequency range of interest.

Even with this much improvement, the detection sensitivity of such an acoustic sensor is still limited by nonthermal noise many score db above thermal noise. Consequently, this Memorandum considers further signal-to-noise ratio improvements achievable by phased array methods. For the purpose of the following discussion, an individual sensor is understood to consist of a single microphone with its associated pipe array; most of 20 db of turbulent noise suppression has already been achieved in the single sensor, and arrays of such sensors are contemplated. The individual sensors are assumed to be omnidirectional below 1 cps.

II. ARRAY GAIN

If some number of sensors are disposed over a region which may be several acoustic wavelengths across, signals from these can be summed in order to achieve still more improvement in the signal-to-turbulent-noise ratio. If signals from N such sensors are summed, most of 10 log N db further improvement should be achieved, the exact amount depending upon the distribution of turbulent scale. However, it is necessary to introduce appropriate time delays in the individual sensor signals before summing, corresponding to the acoustic transit time (and direction) across the array, and the design of a phased array becomes of concern.

Such an array not only serves to reduce turbulent noise, but also exhibits gain. "Gain," of course, indicates a measure of the degree to which the phased array also serves to discriminate against noise which arrives acoustically from distant sources. In practice the signal-to-turbulent-noise ratio is usually poorer in a single sensor, even with a noise-cancelling pipe array, than the signal-to-acoustic-noise ratio--but probably not by a great many db. Consequently, a further improvement of the signal-to-turbulent-noise ratio obtained by a phased array might be vitiated by acoustic noise were it not that the array gain also serves to reduce acoustic noise. The number of db by which a phased array serves to improve the signal-to-noise ratio is not the same for the two noises, since one improvement depends upon the distribution of turbulent scale and the other depends upon the phasing properties of the array. However, the two improvements may turn out to be comparable, in which case the ratio of turbulent

noise to acoustic noise may be about the same in a phased array as it is in a single sensor.

In practice, the desired acoustic signals and the unwanted acoustic noise travel essentially horizontally across the terrain; relatively little acoustic input arrives by steep angles from above. Thus it would be "unfair" to count as an improvement any array discrimination against noises arriving vertically. For this reason the "gain" discussed herein is normalized on 2π radians rather than on 4π steradians--a departure from the "gain commonly discussed in radar and sonar practice. Formally, the term "gain" as used here is defined as follows:

Let there be, at large distance at azimuth φ, a sinusoidal source, frequency f, such that this source produces, in a single sensor, an output of unit amplitude. Let $S(\theta, \varphi, c, c_o, f)$ be the output amplitude from the whole phased array of N sensors, caused by this same source, when the array is phased to azimuth θ on the assumption that the local sound speed is c_o although the correct value is c. Then

$$\text{gain} = \frac{S^2(\theta, \theta, c, c_o, f)}{\frac{1}{2\pi}\int_o^{2\pi} S^2(\theta, \varphi, c, c_o, f)\,d\varphi}$$

= a function of N, the array geometry, θ, f, c, and c_o

III. ARRAY CONFIGURATION

An array might, of course, be assembled in any configuration, and perhaps certain particular arrangements would be advantageous. Indeed, if all azimuths are equally interesting, then circular arrays would seem to be indicated. However, real estate isn't ordinarily available in circular patches. Consequently, arrays disposed in a square region have been chosen for examination.

For the purpose of illustration arrays disposed in a 1/2 x 1/2 statute mile square are considered. This seems to be a reasonable size for detecting nuclear bursts. However, it should be noted that the curves shown below can be shifted to any other size of square merely by shifting the frequency scale so as to preserve the geometry in wavelength measure.

Of the many possible arrangements of sensors within this square, consideration is limited to the set of 4, 9, 16, 25, ..., sensors disposed on the lattice points. This set may not be optimu especially if it should be desirable to discriminate specifically in favor of or against certain fixed azimuths, but it serves adequately to illustrate the amount of array gain achievable with various numbers of sensors. Inasmuch as gain is of interest, rather than side lobe control, shading schemes are not considered.

IV. GAIN VERSUS FREQUENCY FOR SEVERAL ARRAYS

The limiting case of this set of arrays would be a continuously sensitive square, consisting of an infinite number of infinitesimally small sensors, each one properly phased and then all summed. This case furnishes the limit which various arrays of finite numbers of sensors approach, and it serves as a useful reference against which to intercompare the other arrays.

The solid curve of Fig. 1 shows the array gain (in db) versus frequency for such a continuously sensitive 1/2 x 1/2 mile square phased to 0^{o} azimuth (throughout the Memorandum azimuths are measured from the normal to one edge of the square). Of course, the directivity pattern of the square, and thus the array gain, varies with the azimuth to which it is phased; the dotted curve in Fig. 1 shows the gain versus frequency for the same square array phased to 45^{o}. It is seen that the two curves differ negligibly (the variation of gain with azimuth is greater for the finite arrays considered below). In Fig. 1 it is assumed that the velocity of sound is uniform over the array and is known exactly ($c = c_o$ = 1100 ft/sec), so no error in phasing occurs. The solid curve of Fig. 1 represents a more or less ideal case; it is repeated, for reference, in Figs. 2, 3, and 4.

In practice the local velocity of sound, c, is not likely to be known exactly, so as to permit exact phasing. Indeed, it would be expensive and difficult to measure it and to adjust the phasing network as c changes. It would be preferable to be able to assume a nominal local velocity, c_o, phase the signals on this basis, and then accept the degradation of gain which may result if the true velocity

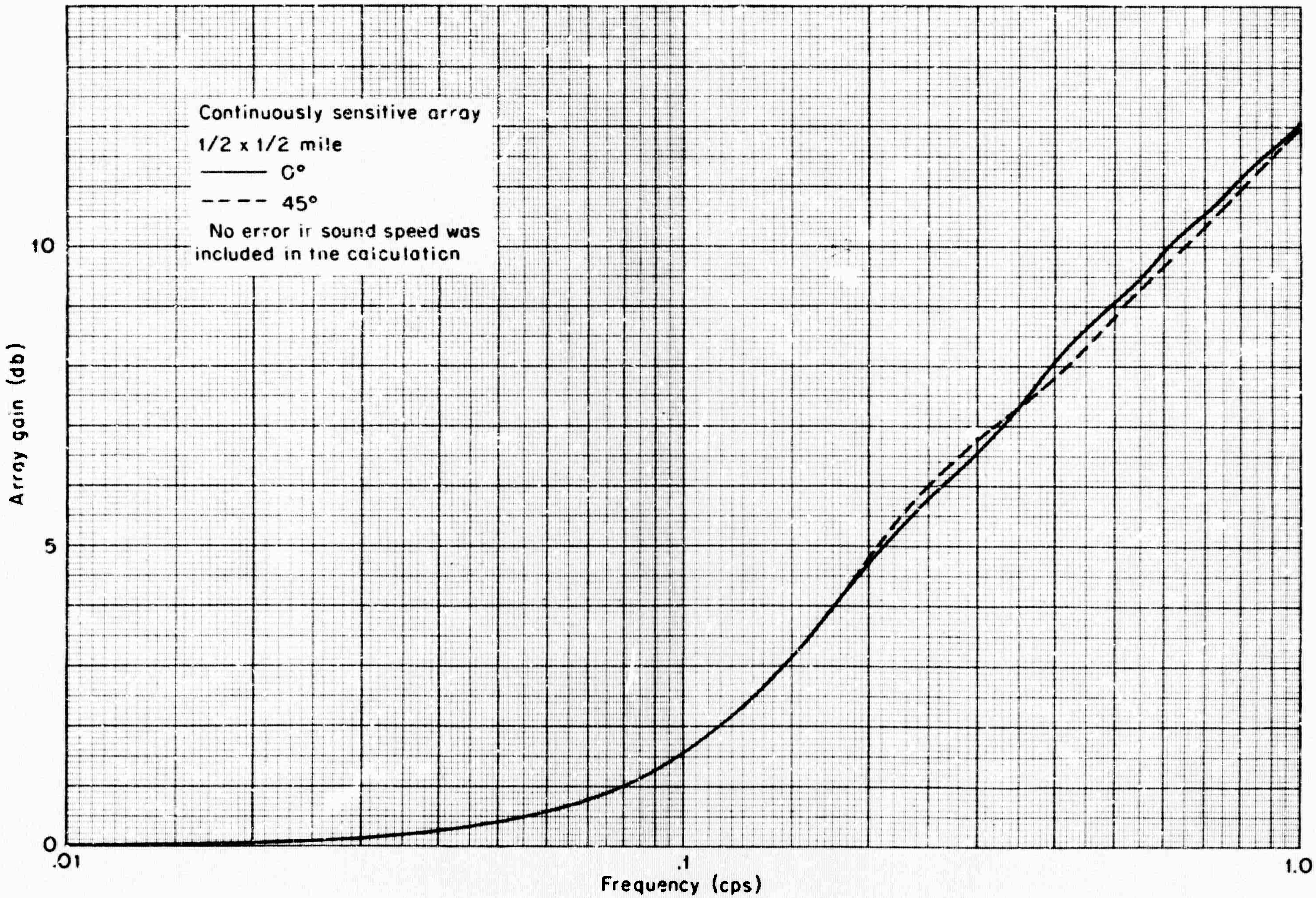

Continuously sensitive array
1/2 x 1/2 mile
C°
45°
No error in sound speed was included in the calculation
Array gain (db)
10
5
0
.01
.1
1.0
Frequency (cps)

Fig. 1

of sound is different from nominal. Consequently, it is not only the ideal gain, when there is no error in sound speed, which is of interest here, but also the sensitivity of gain to error in sound speed.

Furthermore, the dependence of gain upon frequency may be quite different at different azimuths, and so too may the dependence of gain on error in sound speed. There would be little merit in achieving a high gain at some particular azimuth if the same array yielded much poorer gain at another azimuth or at a slightly different sound speed. Consequently, the envelope of maximum and minimum gain achieved, versus frequency, over the range of all possible azimuths and, simultaneously, over a range of sound speeds is of greatest interest. It is this envelope which is shown in Figs. 2, 3, and 4 for arrays of 4, 9, and 16 sensors. On each of these the solid curve of Fig. 1 is repeated, and Figs. 3 and 4 also repeat, dotted, the envelopes shown in the figures which proceed them; thus Fig. 4 permits an intercomparison of all the curves from the previous figures.

To construct these curves, the arrays were assumed to be phased on the basis of a local sound speed c_o = 1100 ft/sec. The gain was computed, at each frequency, for every combination of c = 1000(20) 1200 ft/sec, azimuth = 0(1)45 degrees; at each frequency the maximum and minimum gains are plotted. The mathematical expressions involved are given in the Appendix.

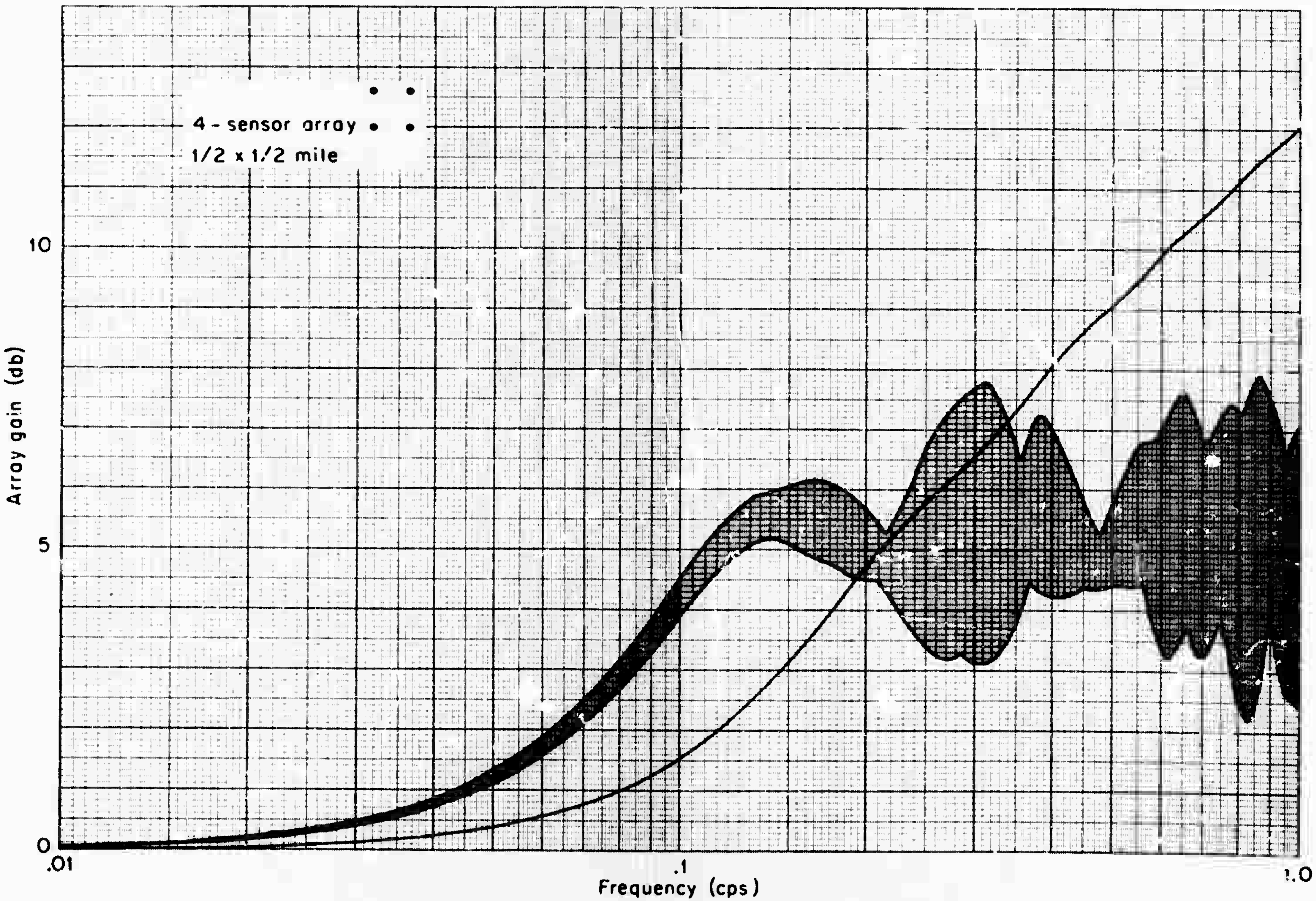

4 - sensor array
1/2 x 1/2 mile
Array gain (db)
10
5
0
.01
.1
1.0
Frequency (cps)

Fig.2

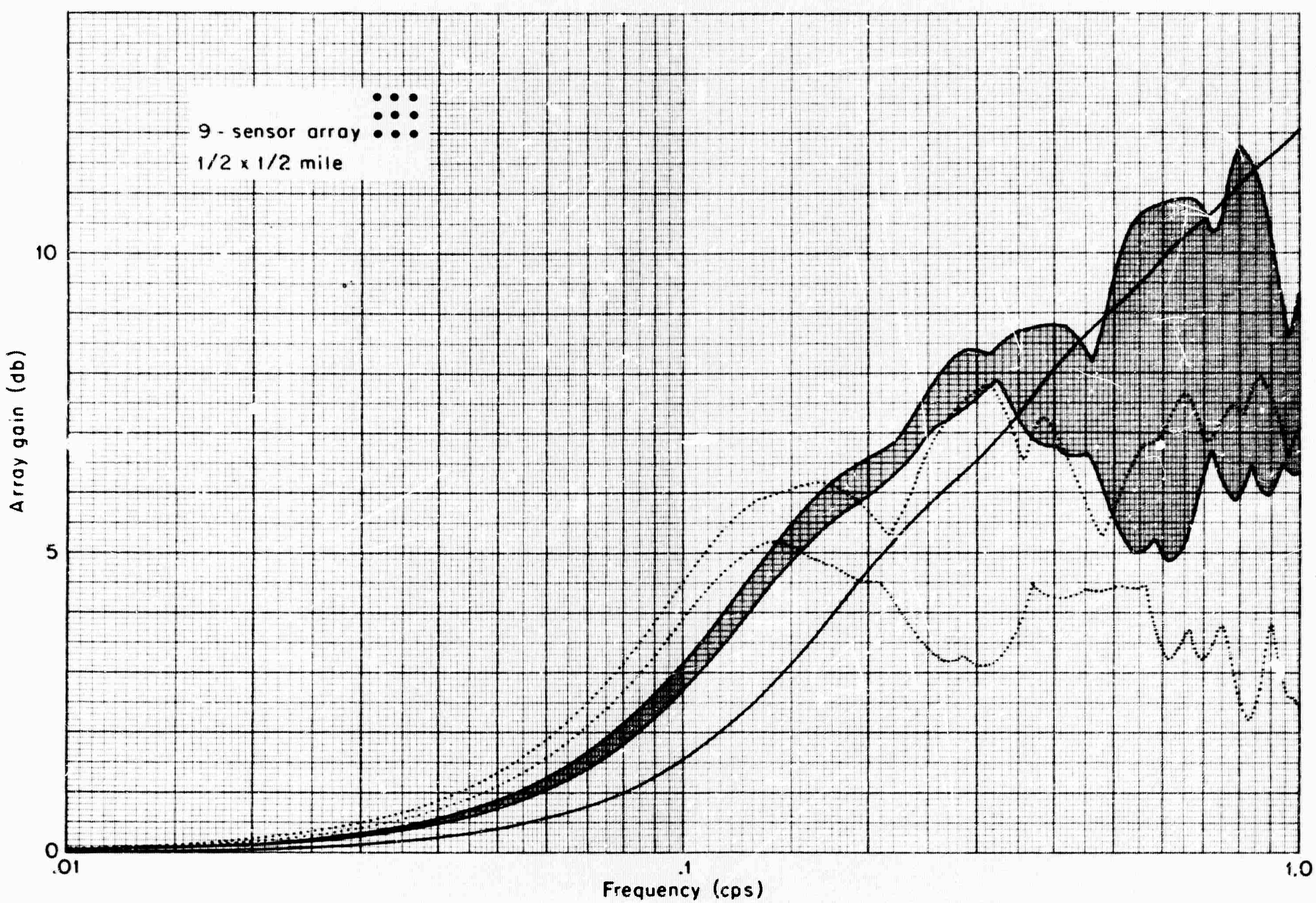

9 - sensor array
1/2 x 1/2 mile
Array gain (db)
10
5
0
.01
.1
1.0
Frequency (cps)

Fig. 3

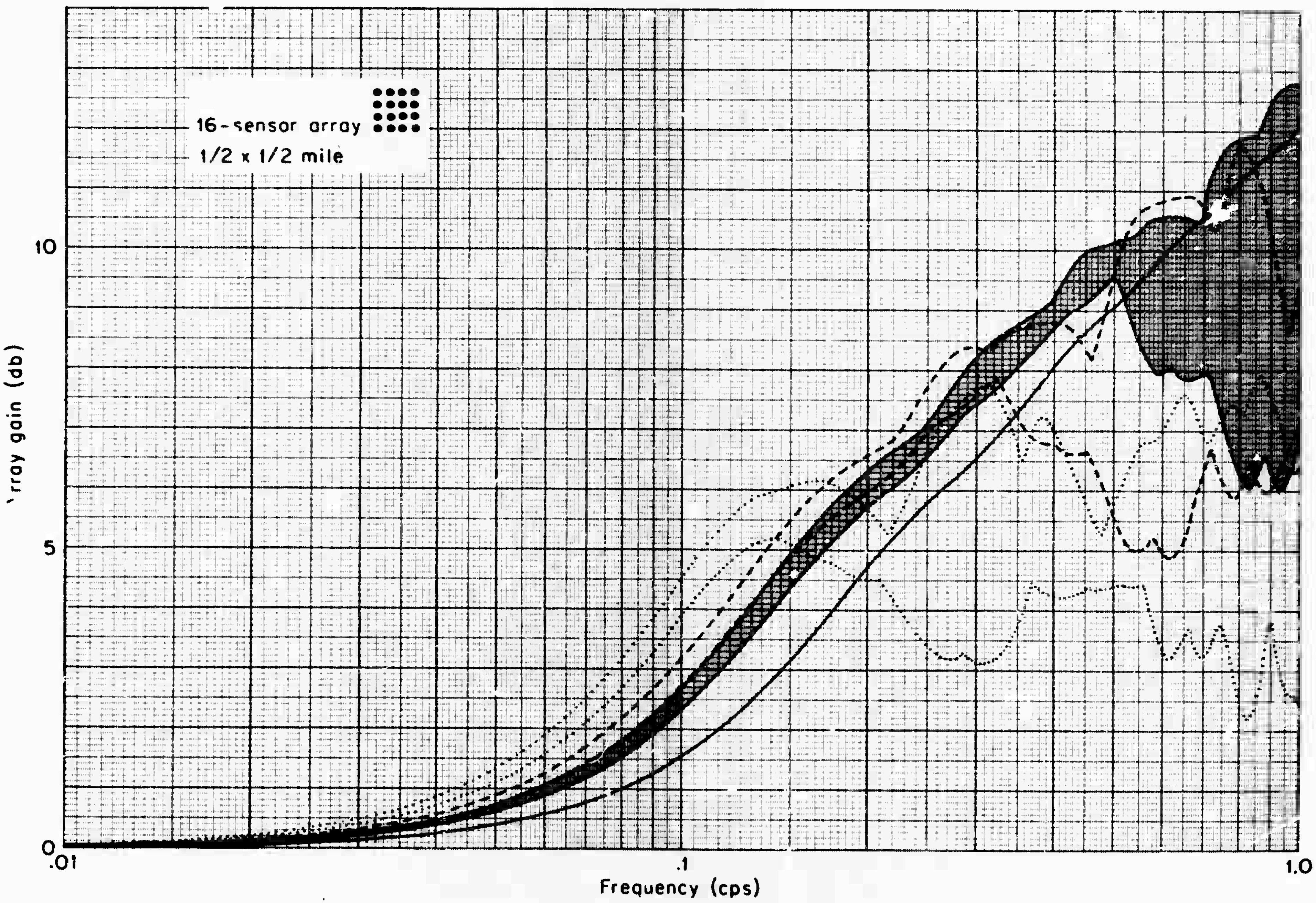

16-sensor array
1/2 x 1/2 mile
Array gain (db)
10
5
0
.01
.1
1.0
Frequency (cps)

Fig.4

V. DISCUSSION OF ARRAY GAINS

For $c = c_o$, and at any one azimuth, the gains of the discrete arrays begin at 0 db at low frequency, curve upward to a slope of approximately 3 db/octave, more or less following the curve for the continuously sensitive array, and then roll over to approach their respective high-frequency asymptotic values (gain = N). It is interesting that, in the low- and middle-frequency regions, the fewer the number of sensors, the higher the gain.

These curves approach their asymptotes quite slowly; even at a single azimuth, and with no error in sound speed, the gain fluctuates quite erratically about the asymptote. The expressions for gain contain Bessel functions of order zero; thus these terms tend to die out asymptotically as $(\text{frequency})^{-1/2}$.

For these discrete arrays it is seen that the variation of gain with azimuth and with sound speed is slight up to a frequency at which the envelope abruptly blooms open to a rather extreme variation. The lower boundary of gain reaches a maximum when the distance between sensors is about 0.4 wavelengths (about 0.33 λ for 4 sensors), and at higher frequencies this lower boundary varies erratically downward. (Because of the error in phasing caused by error in sound speed, the lower boundary goes to negative gain at higher frequencies.) If the frequency at which the lower boundary is a maximum, is adopted as the maximum usable frequency, then:

Number of Sensors	Maximum Usable Frequency (cps)	Minimum Gain at Maximum Usable Frequency (db)
4	.14	5.16
9	.32	7.87
16	.50	9.54

These may be looked upon as "assured" values of gain--the array will yield at least this much at this frequency at any azimuth and despite an error in sound speed as great as that considered ($\pm$ 100 ft/sec). These "assured" values fall short of the "asymptotic" values (6.02, 9.54, and 12.04 db) by an amount which becomes progressively larger as the number of sensors is increased. Thus, for example, quadrupling the number of sensors from 4 to 16 yields only 4.38 db instead of the 6.02 db which might have been hoped for. Clearly the gain tends to improve slowly with increase in the number of sensors; unless steps are taken to measure the sound speed, and thus to reduce the major cause of loss of gain, the maximum practical number of sensors to include in such an array probably does not exceed 16.

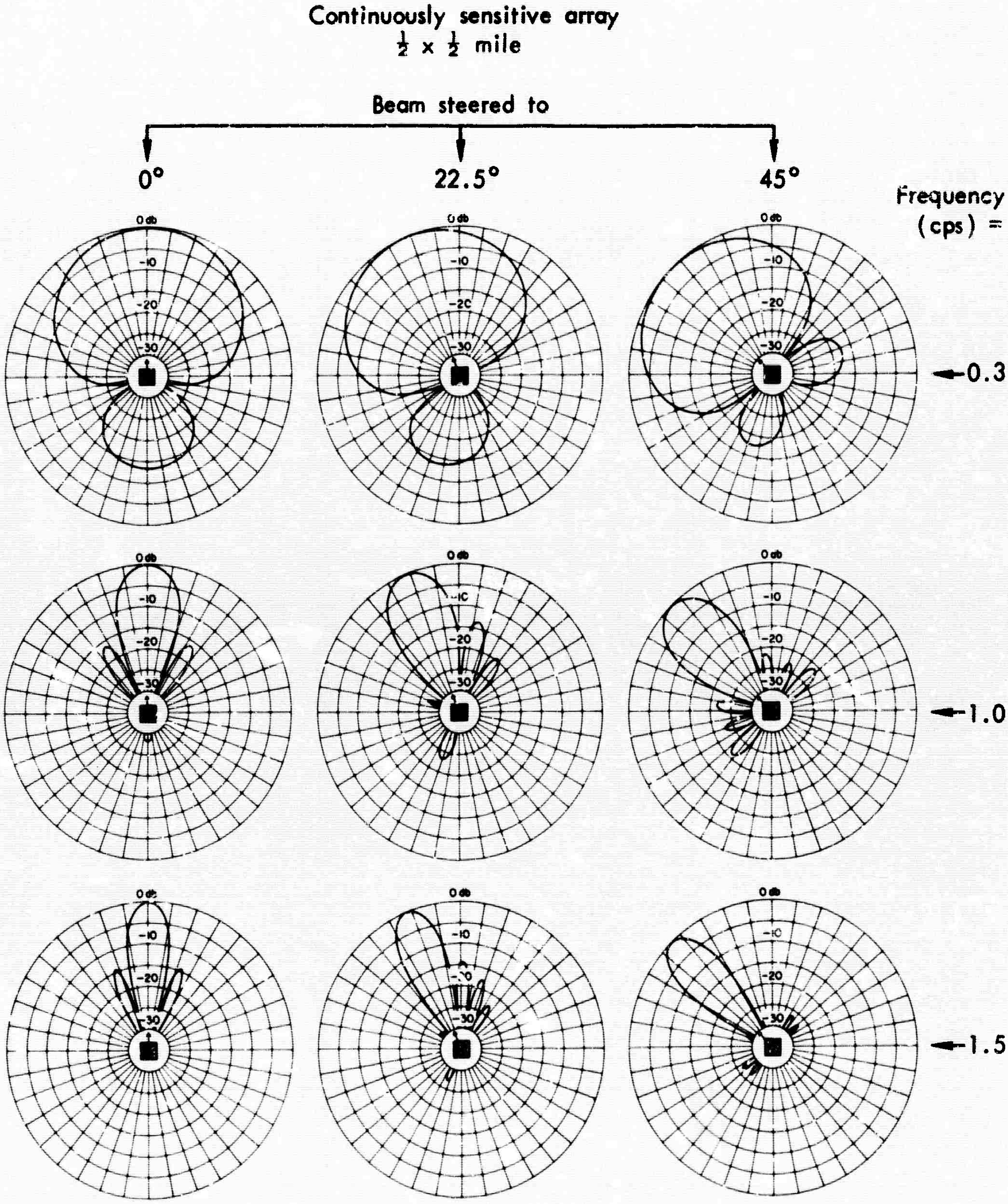
However, the negative aspect of the foregoing remark should not obscure the salient fact that such arrays offer appreciable gain--up to almost 10 db--and this much gain implies a very significant improvement in the range at which a given yield can be detected. Furthermore, the gain rises with frequency--the direction which favors the detection of small yields.

VI. DIRECTIVITY PATTERNS

The purpose of employing arrays such as these is to achieve signal-to-noise ratio improvement (for both turbulent and acoustic noise) so as to improve the detection of weak signals. It is not the purpose to use such phased arrays to determine the azimuth of the signal; indeed these arrays would be relatively poor for this purpose, and azimuth usually can be determined better by correlation methods. However, the directivity patterns of these arrays are of some interest, not only to see why they are not very good for azimuth determination, but also in order to judge how many phasing azimuths are needed to cover 2π radians.

Figures 5-8 show sample directivity patterns of the continuously sensitive array and of the 4-, 9-, and 16-sensor arrays. In these figures no error in sound speed is introduced; such an error would lead to even more irregular patterns, especially at the higher frequencies. For each array, nine patterns are shown, for phasing to 0°, 22.5°, and 45° azimuths, and for frequencies 0.3, 1.0, 1.5 cps. Ordinarily a frequency of 1.0 cps is regarded as the upper end of the band of interest; 1.5 cps is included in these figures as a matter of interest to show how the patterns behave at higher frequency. It will be seen that side lobe heights vary with both frequency and steering azimuth.

As antenna patterns go, these are poor; the side lobes are numerous and high--many about as high as the main beam. The main beam is relatively blunt and not capable of very precise azimuth determination, even apart from the possible ambiguities of side lobe



Continuously sensitive array
½ x ½ mile
Beam steered to
0°
22.5°
45°
Frequency
(cps) =
0.3
1.0
1.5
0 db
-10
-20
-30

Fig.5

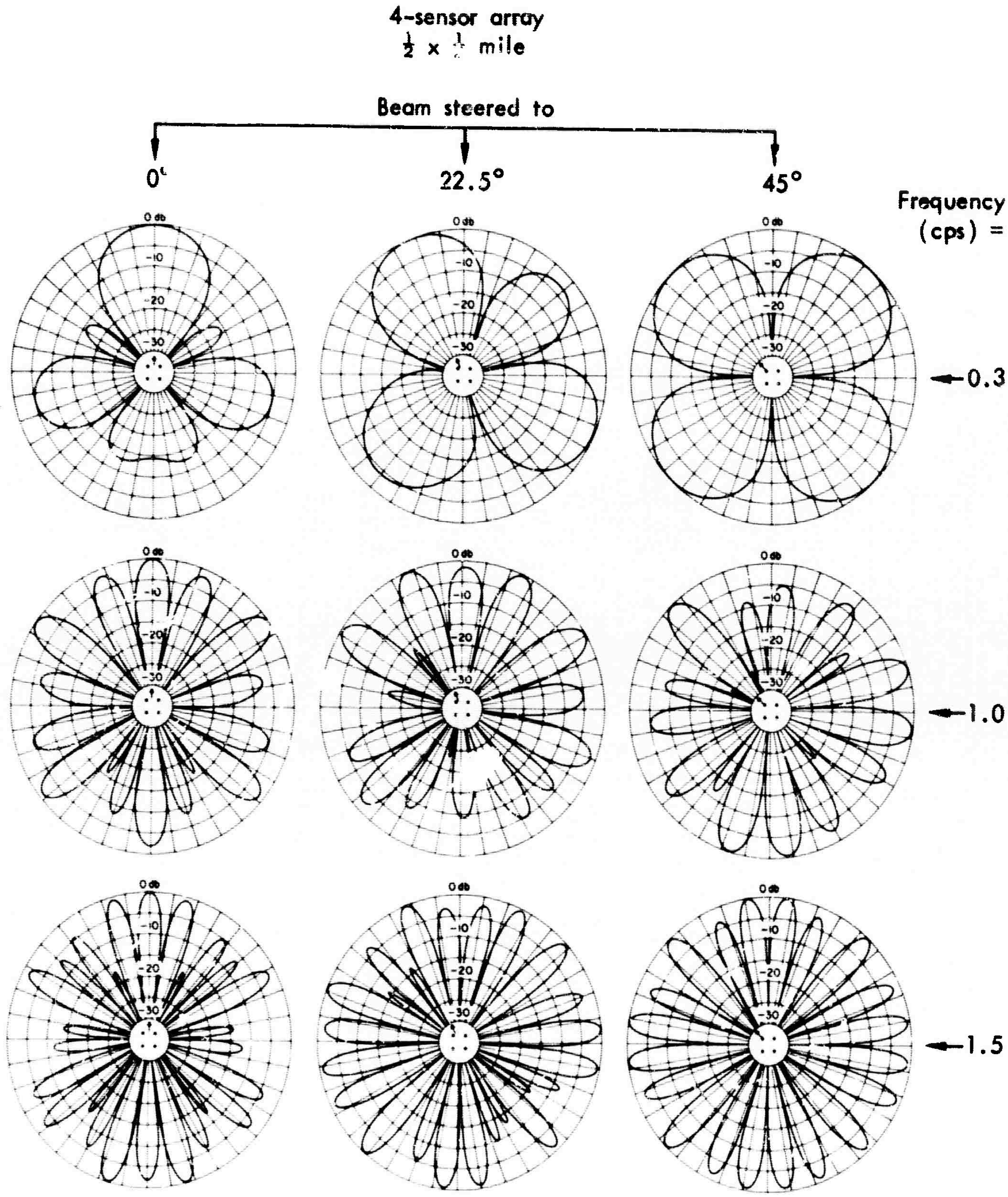

4-sensor array
½ x ½ mile
Beam steered to
0°
22.5°
45°
Frequency
(cps) =
0.3
1.0
1.5
0 db
-10
-20
-30

Fig. 6

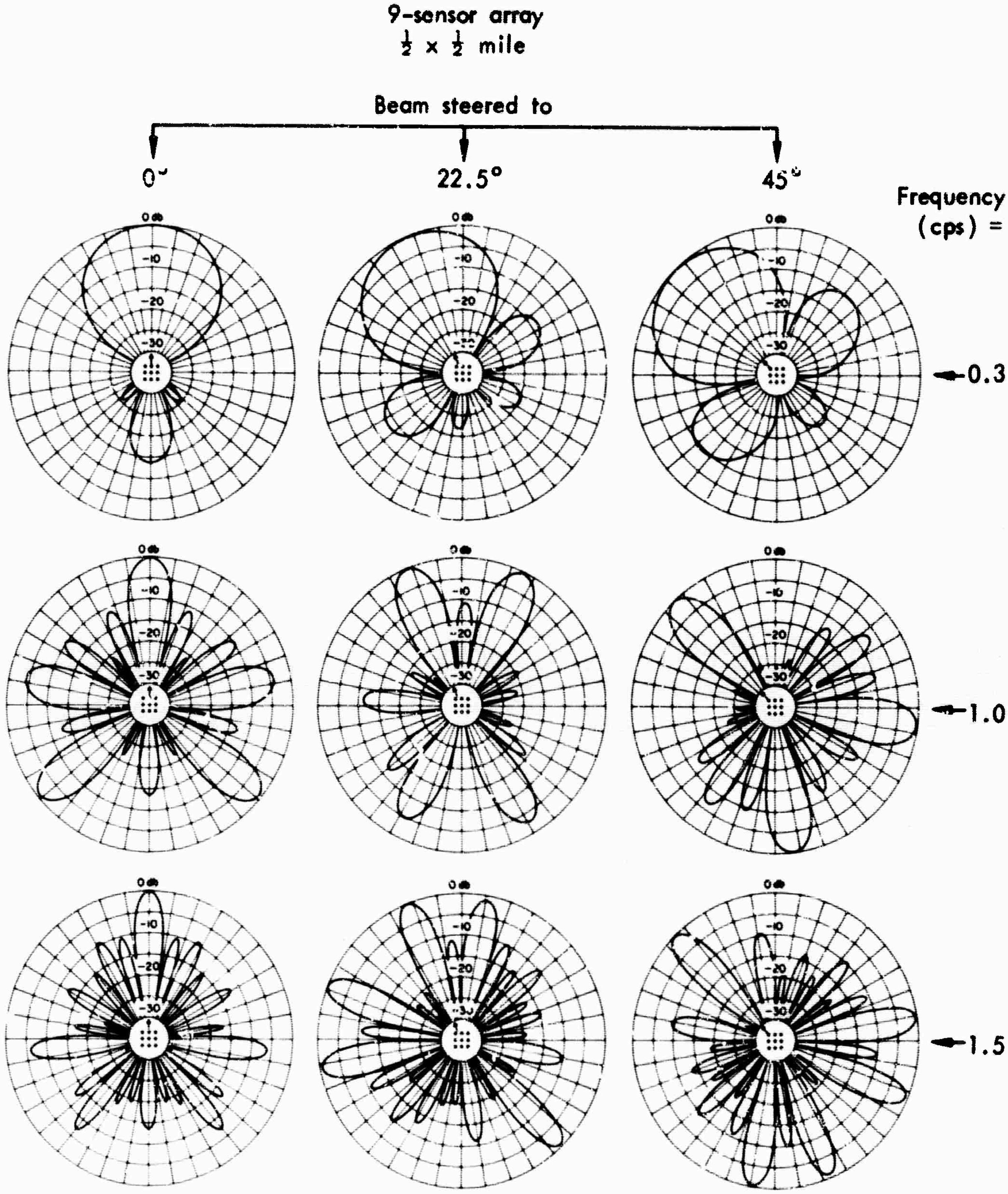

9-sensor array
½ x ½ mile
Beam steered to
0°
22.5°
45°
Frequency (cps) =
0.3
1.0
1.5
0 db
-10
-20
-30

Fig. 7

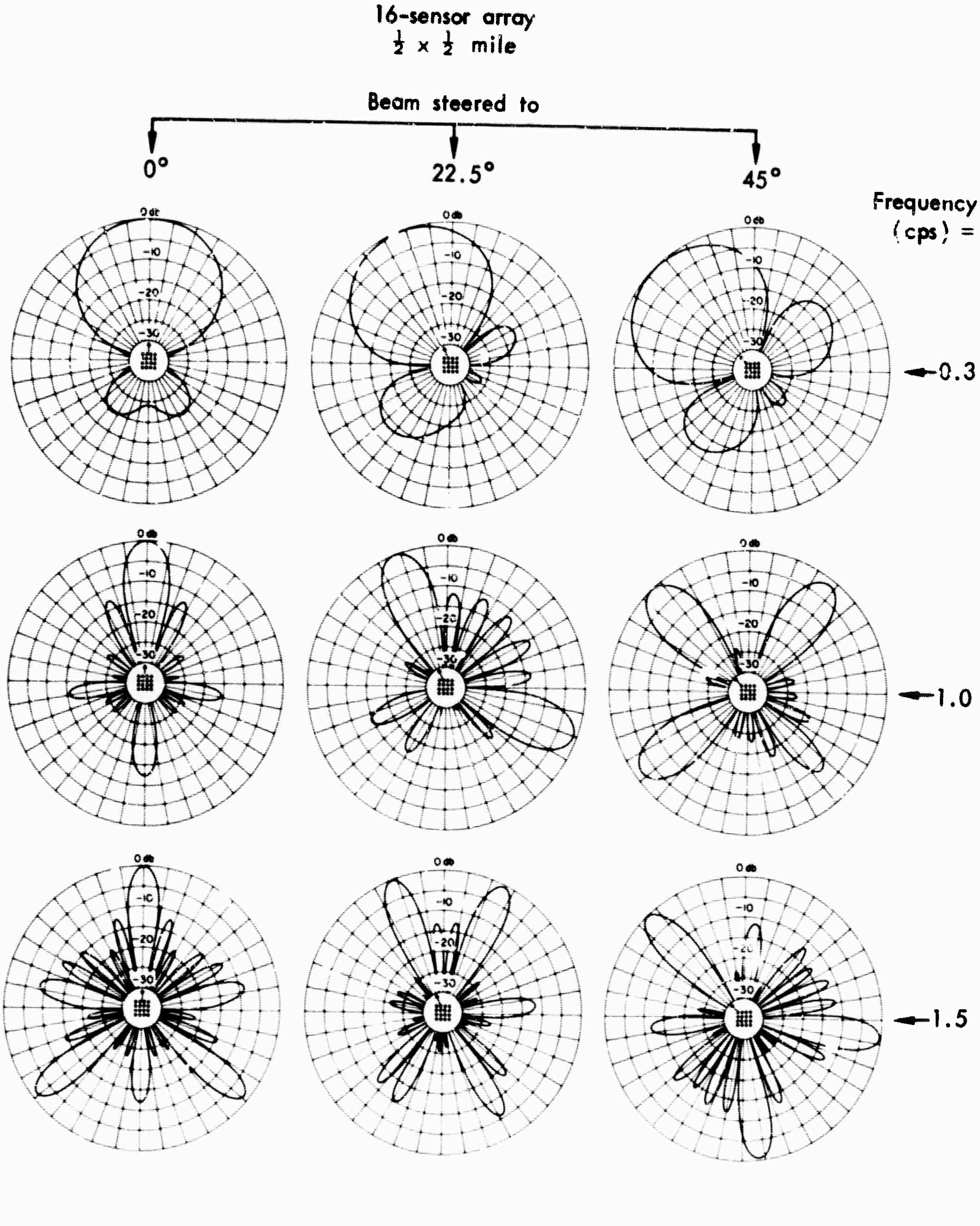

16-sensor array
½ x ½ mile
Beam steered to
0°
22.5°
45°
Frequency (cps) =
0.3
1.0
1.5
0 db
-10
-20
-30

Fig.8

reception. Perhaps it should be emphasized that, poor though these be, they do exhibit gain.

Inasmuch as an array which is phased to some particular azimuth can "cover" adequately only a modest range of angles to each side, it is necessary to arrange, one way or another, to phase the array to a variety of azimuths in order to cover the full 2π radians. This might be done by looking, in time sequence, in one direction after another, or it might be done by looking simultaneously in numerous directions. The merits and drawbacks of these methods are not part of this discussion; in any case it is important to know in how many different directions it is necessary to look.

Figure 9 shows the dependence of the main beam crossover points upon number of beams used at a frequency of 1 cps (the customary upper frequency limit). These curves are based upon the beamwidth for the array phased to azimuth = 0°; the actual crossover points would differ slightly from beam to beam because of the slight change in beamwidth with azimuth. Furthermore, Fig. 9 assumes no error in sound speed; an error would alter these values, perhaps appreciably at certain azimuths. However, this figure provides a sufficient basis upon which to estimate the number of beams needed. The number needed depends upon the crossover value which is accepted and upon the upper frequency to be covered; tabulating the number of beams needed versus the crossover point at 1 cps:

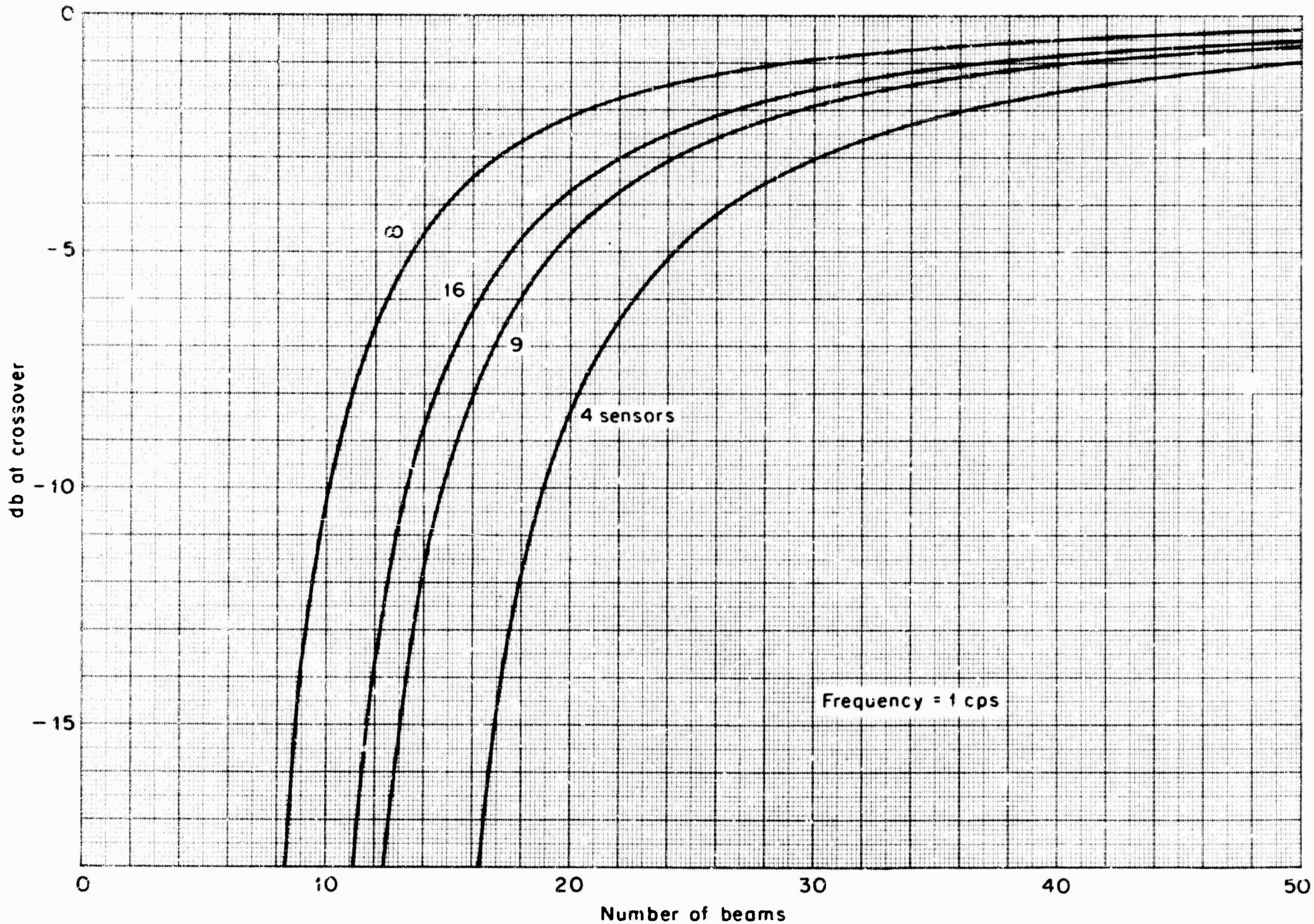

0
-5
-10
-15
db at crossover
0
10
20
30
40
50
Number of beams
∞
16
9
4 sensors
Frequency = 1 cps

Fig. 9

Crossover, db	Number of Beams to Cover 2π 4 sensors	9 sensors	16 sensors
-6.	23	18	17
-5.	25	20	18
-4.	27	22	20
-3.	30	24	22
-2.5	33	27	24
-2.	36	29	27

Clearly, the 16-sensor array is slightly advantageous in this respect because its beamwidth is appreciably wider than the beamwidths of the sparser arrays. This advantage, together with the somewhat better gain, probably justifies the added expense of providing 16 sensors rather than 9. One can infer, from the figure, about how many beams would be needed for arrays of still more sensors, or for arrays of different size.

VII. CONCLUSIONS

The use of phased arrays of omnidirectional acoustic sensors (which themselves contain devices to reduce turbulent noise) would provide significant improvement of both the signal-to-turbulent-noise ratio and the signal-to-acoustic-noise ratio as compared with the output of a single such sensor.

The amount of improvement is greatest at the upper end of the frequency band--thus tending to improve the detection of weaker signals from smaller yields relative to large-yield signals. A 16-element array offers, at its best frequency, about 10 db or more of gain at any azimuth even without correcting for sound speed errors of about $\pm$ 100 ft/sec.

To cover the full 2π radians at 1 cps, a 1/2 x 1/2 mile 16-element array necessitates looking at about 22 azimuths (for -3 db crossover).

The directivity patterns are rather poor for azimuth determination. Thus the utility of such an array would stem from the pronounced improvement in sensitivity for first detection of weak signals, not in improved azimuth determination.

Appendix

DIRECTIVITY PATTERNS AND ARRAY GAINS

In this appendix the expressions for the directivity patterns and array gains discussed in the text are listed without derivation.* The notation defined in the text is used.

Let one edge of the square array be of length L.

Let

$$\beta = 2\pi fL/c$$

$$\beta_o = 2\pi fL/c_o$$

Then the directivity patterns (amplitude) for these arrays are:

<u>for N = 4</u>

$$S = 2\left\{\cos\left[\frac{\beta}{2}(\cos\varphi + \sin\varphi) - \frac{\beta_o}{2}(\cos\theta + \sin\theta)\right]\right.$$

$$\left. + \cos\left[\frac{\beta}{2}(\cos\varphi - \sin\varphi) - \frac{\beta_o}{2}(\cos\theta - \sin\theta)\right]\right\}$$

---

*The expressions given assume that the effective acoustic location of each sensor does not shift with frequency. Inasmuch as each sensor has a noise-cancelling pipe array attached to it, this requires that the pipes be symmetrically disposed about each acoustic sensor. This, in turn, dictates that the pipes extend somewhat beyond the boundary of the 1/2 x 1/2 mile region. However, the relative insensitivity of the array gain to errors in c suggests that this requirement of symmetry is more a formality than a practical necessity.

for N = 9

$$S = 1 + 2 \left\{ \cos \left[\frac{\beta}{2} (\cos \varphi + \sin \varphi) - \frac{\beta_o}{2} (\cos \theta + \sin \theta) \right] \right.$$

$$+ \cos \left[\frac{\beta}{2} (\cos \varphi - \sin \varphi) - \frac{\beta_o}{2} (\cos \theta - \sin \theta) \right]$$

$$+ \cos \left[\frac{\beta}{2} \cos \varphi - \frac{\beta_o}{2} \cos \theta \right]$$

$$\left. + \cos \left[\frac{\beta}{2} \sin \varphi - \frac{\beta_o}{2} \sin \theta \right] \right\}$$

for N = 16

$$S = 2 \left\{ \cos \left[\frac{\beta}{2} (\cos \varphi + \sin \varphi) - \frac{\beta_o}{2} (\cos \theta + \sin \theta) \right] \right.$$

$$+ \cos \left[\frac{\beta}{2} (\cos \varphi - \sin \varphi) - \frac{\beta_o}{2} (\cos \theta - \sin \theta) \right]$$

$$+ \cos \left[\frac{\beta}{2} (\cos \varphi + \frac{1}{3} \sin \varphi) - \frac{\beta_o}{2} (\cos \theta + \frac{1}{3} \sin \theta) \right]$$

$$+ \cos \left[\frac{\beta}{2} (\cos \varphi - \frac{1}{3} \sin \varphi) - \frac{\beta_o}{2} (\cos \theta - \frac{1}{3} \sin \theta) \right]$$

$$+ \cos \left[\frac{\beta}{2} (\frac{1}{3} \cos \varphi + \sin \varphi) - \frac{\beta_o}{2} (\frac{1}{3} \cos \theta + \sin \theta) \right]$$

$$+ \cos \left[\frac{\beta}{2} (\frac{1}{3} \cos \varphi - \sin \varphi) - \frac{\beta_o}{2} (\frac{1}{3} \cos \theta - \sin \theta) \right]$$

$$+ \cos \left[\frac{\beta}{2} (\frac{1}{3} \cos \varphi + \frac{1}{3} \sin \varphi) - \frac{\beta_o}{2} (\frac{1}{3} \cos \theta + \frac{1}{3} \sin \theta) \right]$$

$$\left. + \cos \left[\frac{\beta}{2} (\frac{1}{3} \cos \varphi - \frac{1}{3} \sin \varphi) - \frac{\beta_o}{2} (\frac{1}{3} \cos \theta - \frac{1}{3} \sin \theta) \right] \right\}$$

for N = ∞

$$S = \left\{ \frac{\sin\left[\frac{\beta}{2}\cos\varphi - \frac{\beta_o}{2}\cos\theta\right]}{\left[\frac{\beta}{2}\cos\varphi - \frac{\beta_o}{2}\cos\theta\right]} \right\} \cdot \left\{ \frac{\sin\left[\frac{\beta}{2}\sin\varphi - \frac{\beta_o}{2}\sin\theta\right]}{\left[\frac{\beta}{2}\sin\varphi - \frac{\beta_o}{2}\sin\theta\right]} \right\}$$

As to array gain, the numerator of the gain expression poses no problem; it is only necessary to set $\varphi = \theta$ in the foregoing pattern expressions, and then procede to evaluate the numerator. The difficulty resides in the evaluation of the denominator. Let the denominator be

$$D = \frac{1}{2\pi} \int_{o}^{2\pi} S^2 \, d\varphi$$

Then, for these arrays:

for N = 4

$$D = 4 + 4 J_o(\beta) \{ \cos[\beta_o \cos\theta] + \cos[\beta_o \sin\theta] \}$$
$$+ 2 J_o(\sqrt{2}\,\beta) \{ \cos[\sqrt{2}\,\beta_o \cos(\tfrac{\pi}{4} - \theta)] + \cos[\sqrt{2}\,\beta_o \sin(\tfrac{\pi}{4} - \theta)] \}$$

for N = 9

$$D = 9 + 6 J_o(\beta) \left\{ \cos\left[\beta_o \cos\theta\right] + \cos\left[\beta_o \sin\theta\right] \right\}$$
$$+ 12 J_o\left(\frac{\beta}{2}\right) \left\{ \cos\left[\frac{\beta_o}{2}\cos\theta\right] + \cos\left[\frac{\beta_o}{2}\sin\theta\right] \right\}$$
$$+ 2 J_o\left(\sqrt{2}\,\beta\right) \left\{ \cos\left[\sqrt{2}\,\beta_o \cos(\tfrac{\pi}{4} - \theta)\right] + \cos\left[\sqrt{2}\,\beta_o \sin(\tfrac{\pi}{4} - \theta)\right] \right\}$$
$$+ 8 J_o\left(\frac{\sqrt{2}}{2}\beta\right) \left\{ \cos\left[\frac{\sqrt{2}}{2}\beta_o \cos(\tfrac{\pi}{4} - \theta)\right] + \cos\left[\frac{\sqrt{2}}{2}\beta_o \sin(\tfrac{\pi}{4} - \theta)\right] \right\}$$

(continued)

$$+ 4 J_o \left(\frac{\sqrt{5}}{2}\beta\right) \left\{ \cos\left[\frac{\sqrt{5}}{2}\beta_o \cos(\alpha_1 - \theta)\right] + \cos\left[\frac{\sqrt{5}}{2}\beta_o \sin(\alpha_1 - \theta)\right]\right.$$

$$\left. + \cos\left[\frac{\sqrt{5}}{2}\beta_o \cos(\alpha_1 + \theta)\right] + \cos\left[\frac{\sqrt{5}}{2}\beta_o \sin(\alpha_1 + \theta)\right]\right\}$$

where $\sin\alpha_1 = \frac{1}{\sqrt{5}}$ $\quad$ $\cos\alpha_1 = \frac{2}{\sqrt{5}}$

<u>for N = 16</u>

$$D = 16 + 8 J_o(\beta) \left\{ \cos\left[\beta_o \cos\theta\right] + \cos\left[\beta_o \sin\theta\right]\right\}$$

$$+ 16 J_o\left(\frac{2}{3}\beta\right) \left\{ \cos\left[\frac{2}{3}\beta_o \cos\theta\right] + \cos\left[\frac{2}{3}\beta_o \sin\theta\right]\right\}$$

$$+ 24 J_o\left(\frac{1}{3}\beta\right) \left\{ \cos\left[\frac{1}{3}\beta_o \cos\theta\right] + \cos\left[\frac{1}{3}\beta_o \sin\theta\right]\right\}$$

$$+ 2 J_o\left(\sqrt{2}\beta\right) \left\{ \cos\left[\sqrt{2}\beta_o \cos\left(\frac{\pi}{4} - \theta\right)\right] + \cos\left[\sqrt{2}\beta_o \sin\left(\frac{\pi}{4} - \theta\right)\right]\right\}$$

$$+ 8 J_o\left(\frac{2\sqrt{2}}{3}\beta\right) \left\{ \cos\left[\frac{2\sqrt{2}}{3}\beta_o \cos\left(\frac{\pi}{4} - \theta\right)\right] + \cos\left[\frac{2\sqrt{2}}{3}\beta_o \sin\left(\frac{\pi}{4} - \theta\right)\right]\right\}$$

$$+ 18 J_o\left(\frac{\sqrt{2}}{3}\beta\right) \left\{ \cos\left[\frac{\sqrt{2}}{3}\beta_o \cos\left(\frac{\pi}{4} - \theta\right)\right] + \cos\left[\frac{\sqrt{2}}{3}\beta_o \sin\left(\frac{\pi}{4} - \theta\right)\right]\right\}$$

$$+ 6 J_o\left(\frac{\sqrt{10}}{3}\beta\right) \left\{ \cos\left[\frac{\sqrt{10}}{3}\beta_o \cos(\alpha_2 - \theta)\right] + \cos\left[\frac{\sqrt{10}}{3}\beta_o \sin(\alpha_2 - \theta)\right]\right.$$

$$\left. + \cos\left[\frac{\sqrt{10}}{3}\beta_o \cos(\alpha_2 + \theta)\right] + \cos\left[\frac{\sqrt{10}}{3}\beta_o \sin(\alpha_2 + \theta)\right]\right\}$$

$$+ 12 J_o\left(\frac{\sqrt{5}}{3}\beta\right) \left\{ \cos\left[\frac{\sqrt{5}}{3}\beta_o \cos(\alpha_1 - \theta)\right] + \cos\left[\frac{\sqrt{5}}{3}\beta_o \sin(\alpha_1 - \theta)\right]\right.$$

$$\left. + \cos\left[\frac{\sqrt{5}}{3}\beta_o \cos(\alpha_1 + \theta)\right] + \cos\left[\frac{\sqrt{5}}{3}\beta_o \sin(\alpha_1 + \theta)\right]\right\}$$

$$+ 4 J_o\left(\frac{\sqrt{13}}{3}\beta\right) \left\{ \cos\left[\frac{\sqrt{13}}{3}\beta_o \cos(\alpha_3 - \theta)\right] + \cos\left[\frac{\sqrt{13}}{3}\beta_o \sin(\alpha_3 - \theta)\right]\right.$$

$$\left. + \cos\left[\frac{\sqrt{13}}{3}\beta_o \cos(\alpha_3 + \theta)\right] + \cos\left[\frac{\sqrt{13}}{3}\beta_o \sin(\alpha_3 + \theta)\right]\right\}$$

where $\sin \alpha_2 = \frac{3}{\sqrt{10}}$ $\cos \alpha_2 = \frac{1}{\sqrt{10}}$

$\sin \alpha_3 = \frac{2}{\sqrt{13}}$ $\cos \alpha_3 = \frac{3}{\sqrt{13}}$

for $N = \infty$

No expression for D has been obtained. The points needed to plot Fig. 1 were obtained by numerical integration on a digital computer. This is a relatively slow and expensive routine, and it was not thought to be worthwhile to explore the full domain of c and θ, as was done for the finite values of N.

Note that these expressions for D have a common form

D = N + a sum of terms of the form

$$n\, J_o(k\beta) \left\{ \cos \left[k\beta_o \cos (\alpha - \theta)\right] + \cos \left[k\beta_o \sin (\alpha - \theta)\right] \right\}$$

It will be seen that n, k, and α are related to the geometry of the array itself as follows:

- Connect every sensor point in the array to every other point. In doing so, lines which are themselves component parts of longer lines are nevertheless to be regarded as distinct.
- Then n = the total number of such lines whose length is kL and which lie at azimuth α in the array.

For example, in the array of 4 sensors, there are four lines (the edges) of length such that $k = 1$ and whose azimuths are either 0 or $\pi/2$ (and these combine because $\cos(\pi/2 - \alpha) = \sin \alpha$), and there are two lines (the diagonals) whose length is such that $k = \sqrt{2}$ and whose azimuths are $\pm \pi/4$ (which also combine). In the 9-element array these same 6 lines are seen, plus two more lines of length corresponding to $k = 1$ passing through the center, plus lines with $k = \sqrt{2}/2$, $k = 1/2$, and $k = \sqrt{5}/2$.

This structure for the expression for D holds for all finite N. The proof of this rule is not difficult.

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10. ABSTRACT

A discussion of the possible use of phased arrays in detecting the acoustic signal from atmospheric nuclear bursts. This Memorandum considers, for simple unshaded square plane arrays, the dependence of array gain upon frequency, azimuth, size of array, number of sensors in the array, and uncertainty in local velocity of sound. The author concludes that the use of phased arrays of omnidirectional acoustic sensors would significantly improve both the signal-to-turbulent-noise ratio and the signal-to-acoustic-noise ratio.

11. KEY WORDS

Detection
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